

Principles of Computer Science II

Abstract Data Types

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Lecture 18



Heap Data Structures

Heap is a special case of balanced binary tree data structure where the root-node key is compared with its children and arranged accordingly.

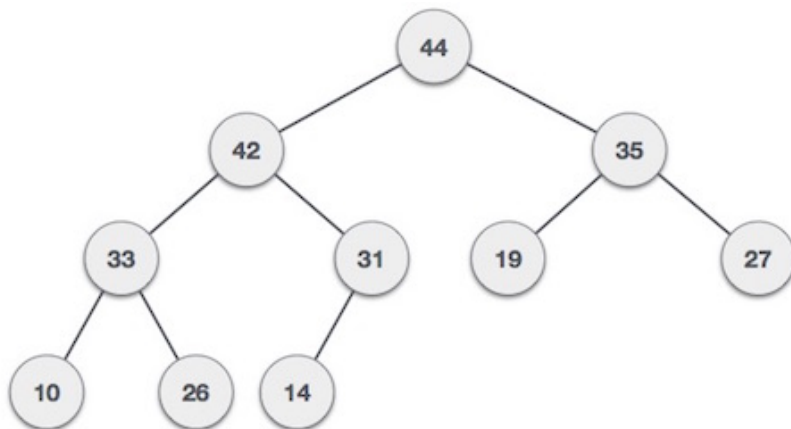
If α has child node β then

$$key(\alpha) \geq key(\beta)$$

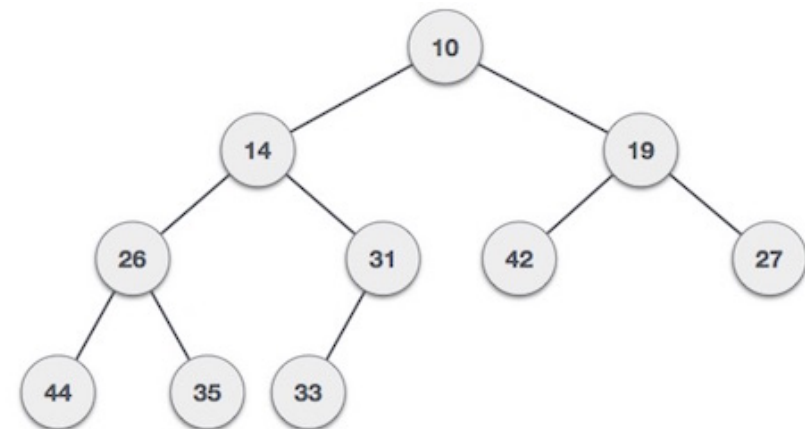
- ▶ As the value of parent is greater than that of child, this property generates **Max Heap**.
- ▶ By changing the criterion we can also get a **Min Heap**.



Max Heap



Min Heap



Max Heap – Add

1. Create a new node at the end of heap.
2. Assign new value to the node.
3. Compare the value of this child node with its parent.
4. If value of parent is less than child, then swap them.
5. Repeat step 3 & 4 until Heap property holds.

Note – In Min Heap construction algorithm, we expect the value of the parent node to be less than that of the child node.



Max Heap – Add Example

Input 35 33 42 10 14 19 27 44 26 31



Max Heap – Add Example

Input **35** 33 42 10 14 19 27 44 26 31

35



Max Heap – Add Example

Input **35** 33 42 10 14 19 27 44 26 31

35



Max Heap – Add Example

Input 35 33 42 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 **33** 42 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 **33** 42 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 **33** 42 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 **33** 42 10 14 19 27 44 26 31



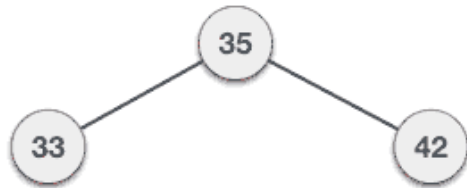
Max Heap – Add Example

Input 35 33 **42** 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 33 **42** 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 33 **42** 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 33 42 10 14 19 27 44 26 31



Max Heap – Add Example

Input 35 33 42 10 14 19 27 44 26 31



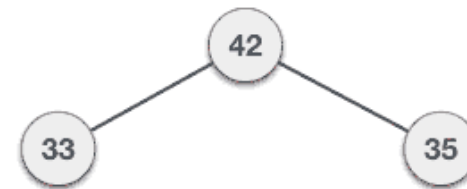
Max Heap – Add Example

Input 35 33 42 10 14 19 27 44 26 31



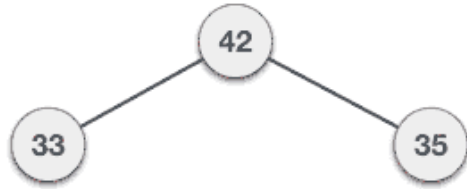
Max Heap – Add Example

Input 35 33 42 10 14 19 27 44 26 31



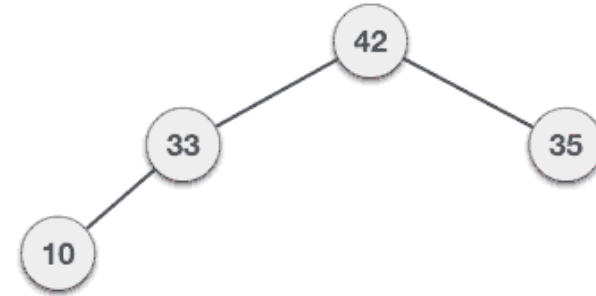
Max Heap – Add Example

Input 35 33 42 **10** 14 19 27 44 26 31



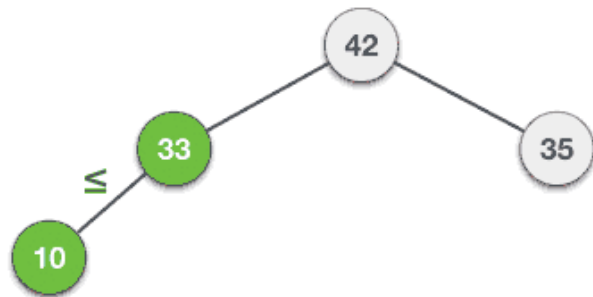
Max Heap – Add Example

Input 35 33 42 **10** 14 19 27 44 26 31



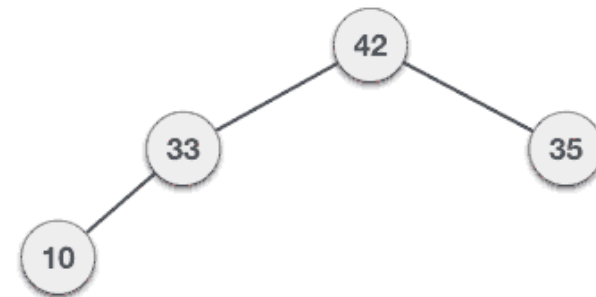
Max Heap – Add Example

Input 35 33 42 **10** 14 19 27 44 26 31



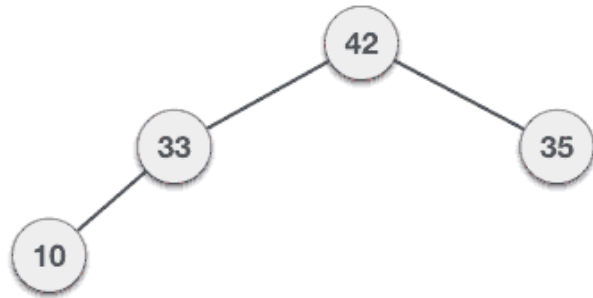
Max Heap – Add Example

Input 35 33 42 **10** 14 19 27 44 26 31



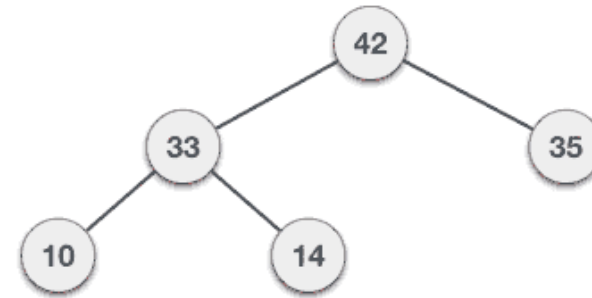
Max Heap – Add Example

Input 35 33 42 10 **14** 19 27 44 26 31



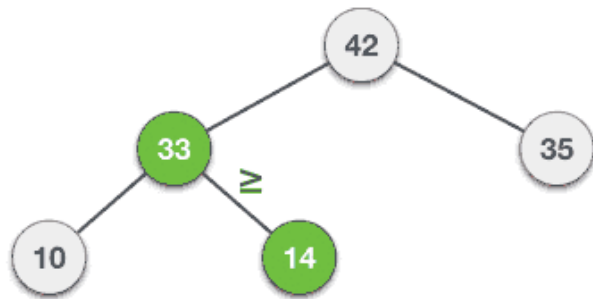
Max Heap – Add Example

Input 35 33 42 10 **14** 19 27 44 26 31



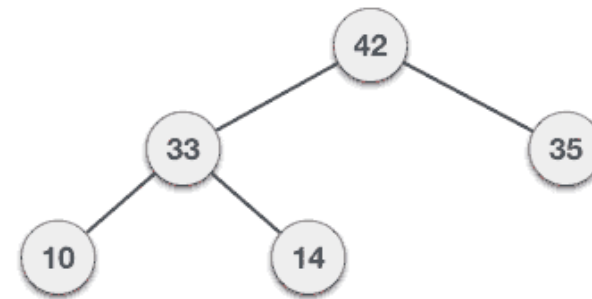
Max Heap – Add Example

Input 35 33 42 10 **14** 19 27 44 26 31



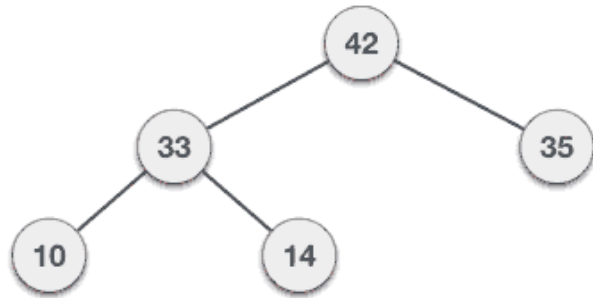
Max Heap – Add Example

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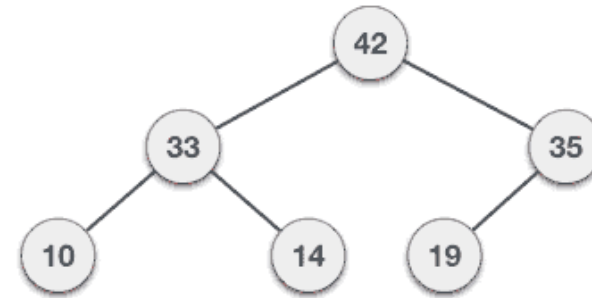
Max Heap – Add Example

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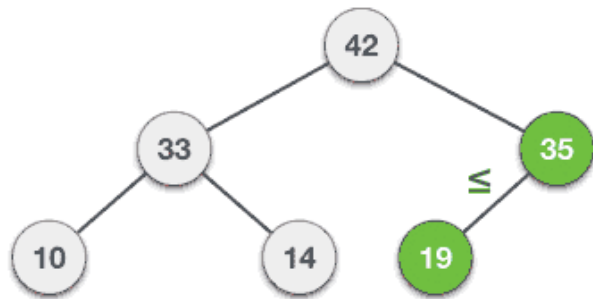
Max Heap – Add Example

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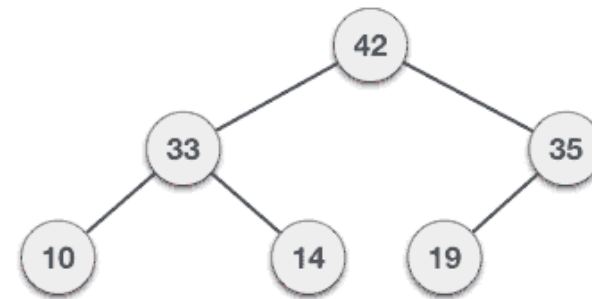
Max Heap – Add Example

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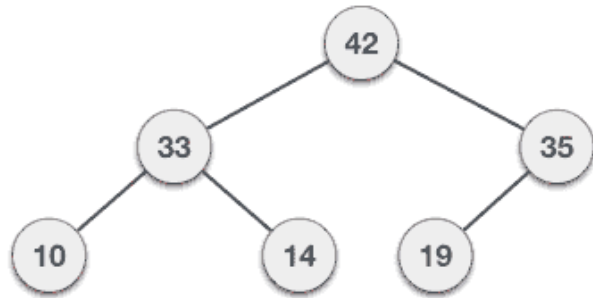
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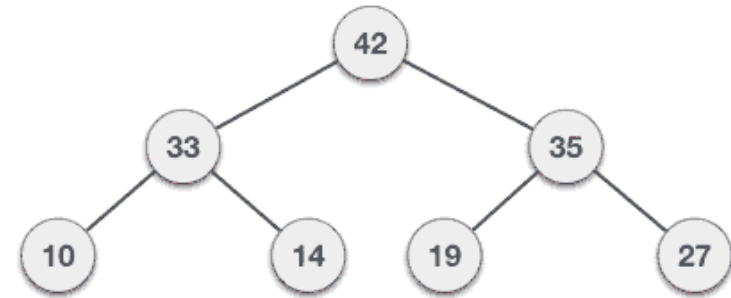
Max Heap – Add Example

Input 35 33 42 10 14 19 **27** 44 26 31



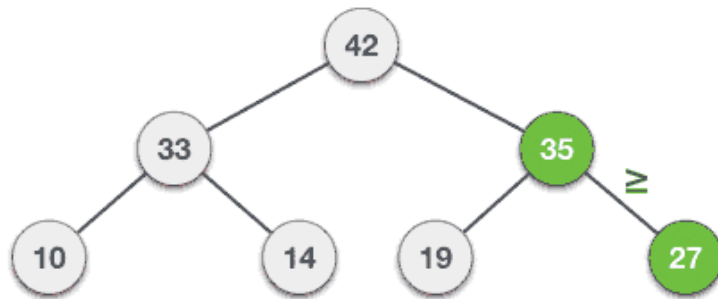
Max Heap – Add Example

Input 35 33 42 10 14 19 **27** 44 26 31



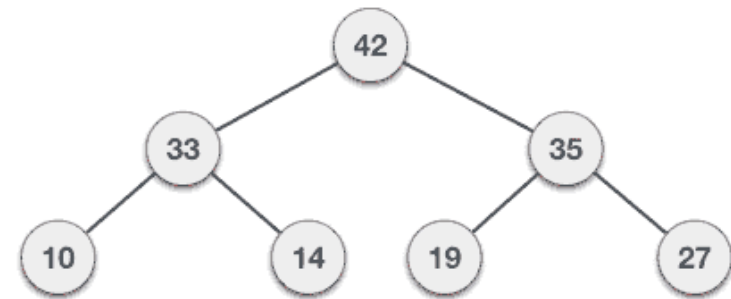
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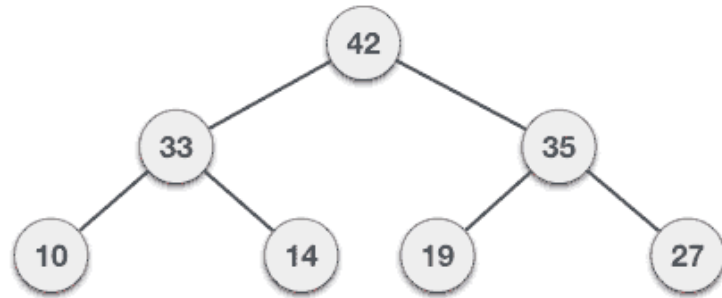
Max Heap – Add Example

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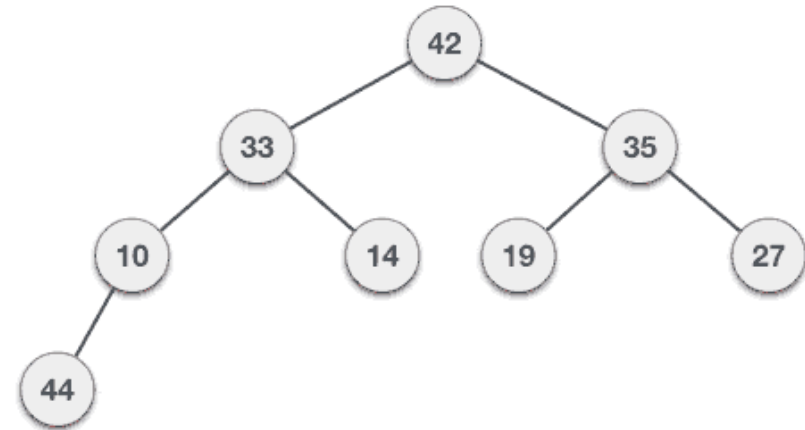
Max Heap – Add Example

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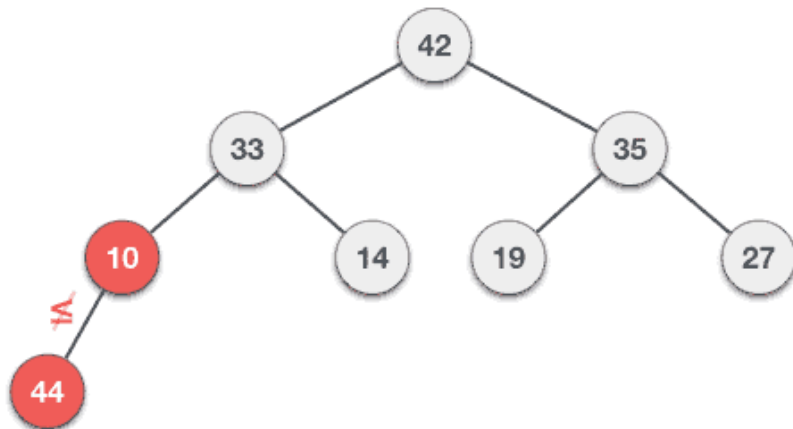
Max Heap – Add Example

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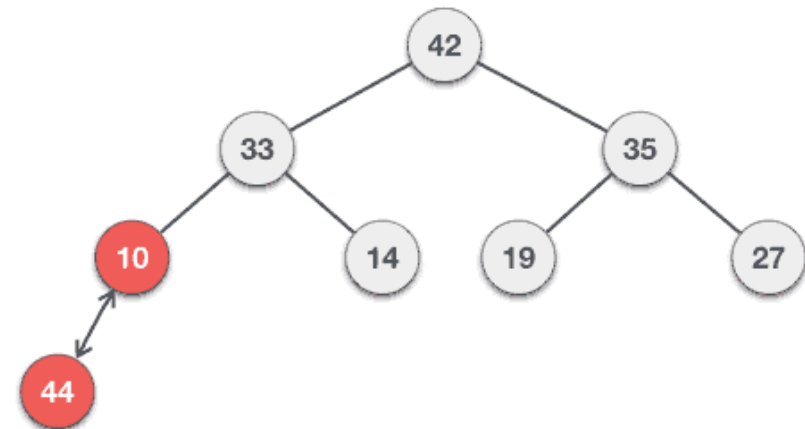
Max Heap – Add Example

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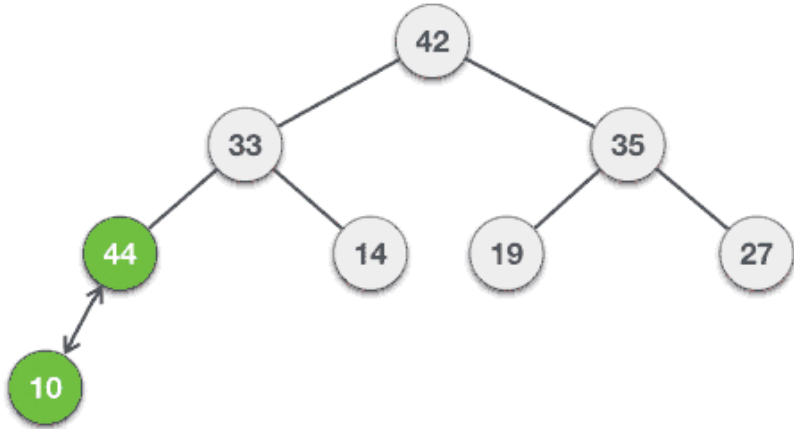
Max Heap – Add Example

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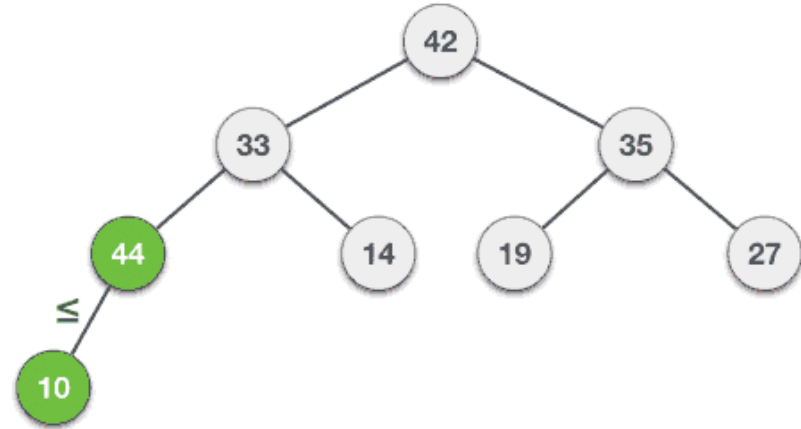
Max Heap – Add Example

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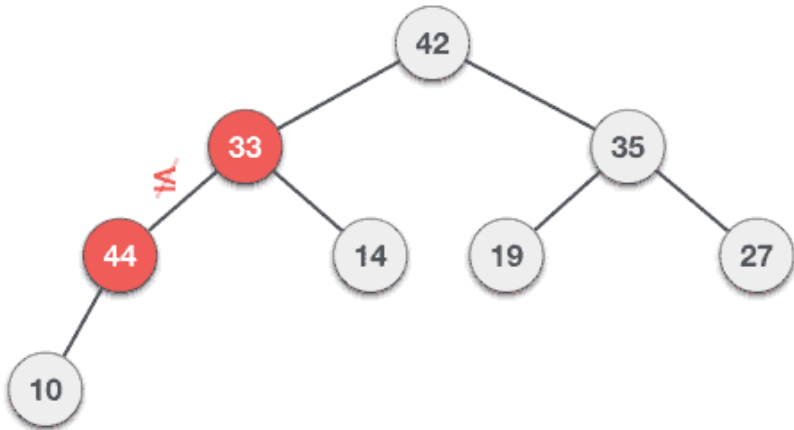
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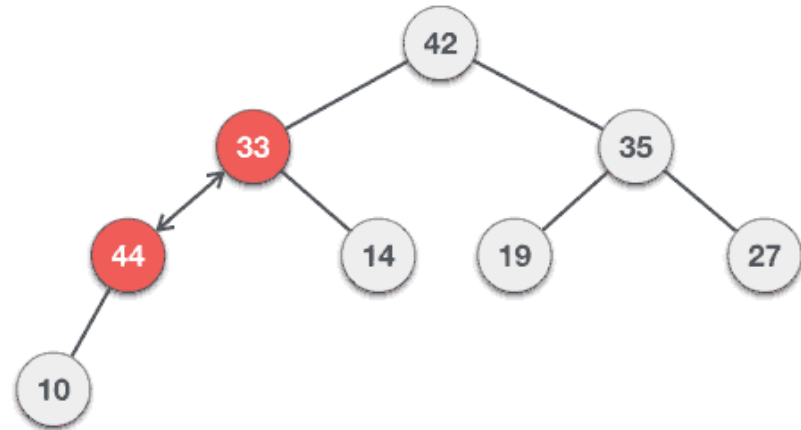
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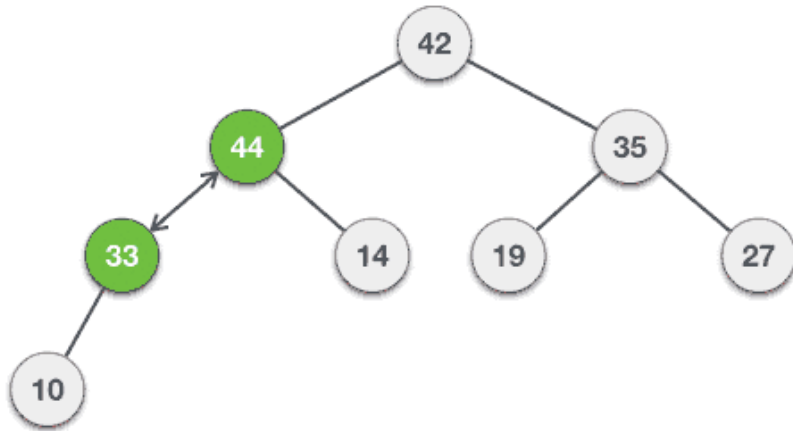
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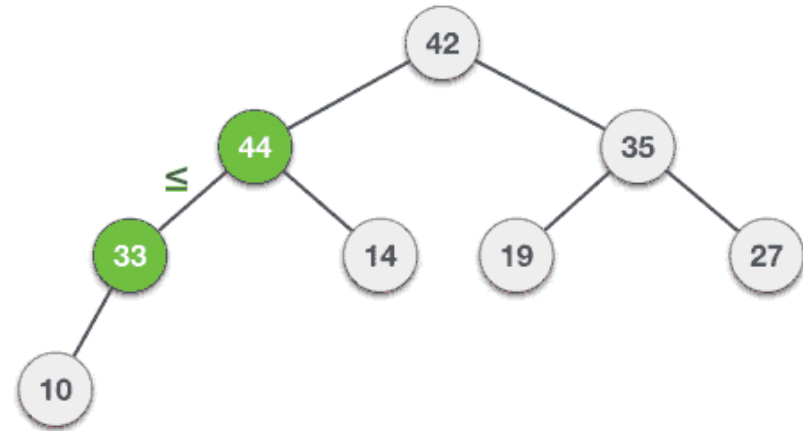
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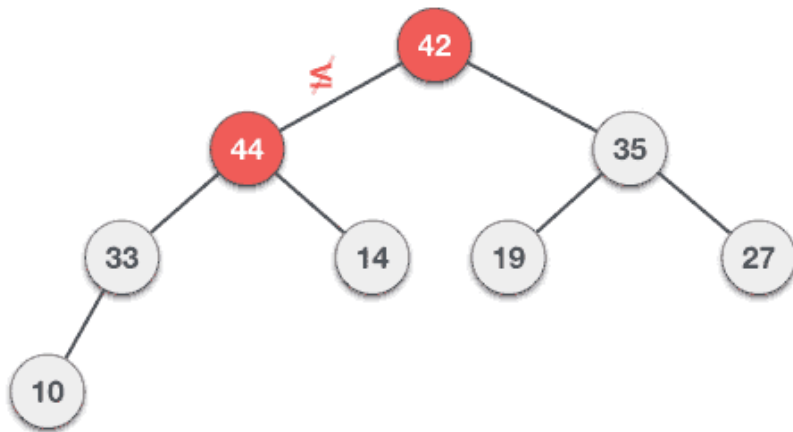
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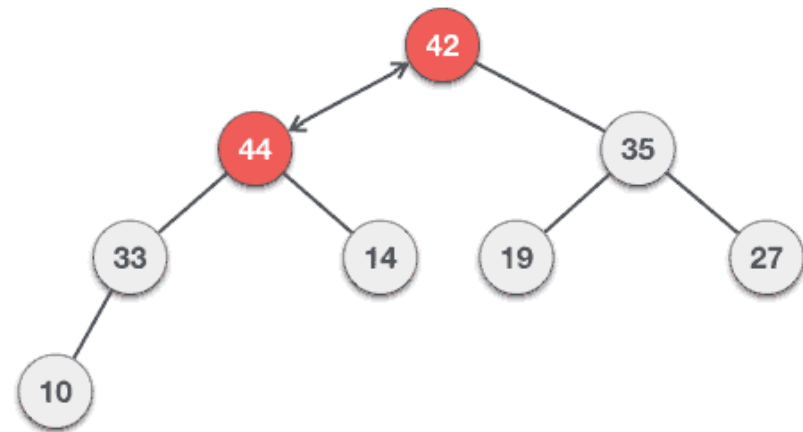
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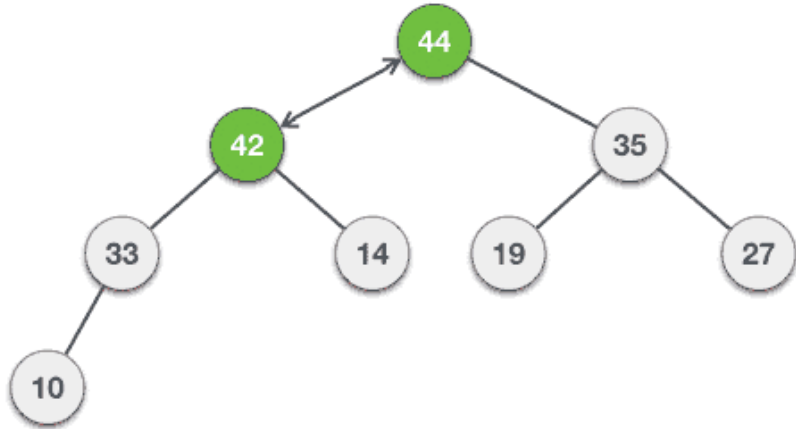
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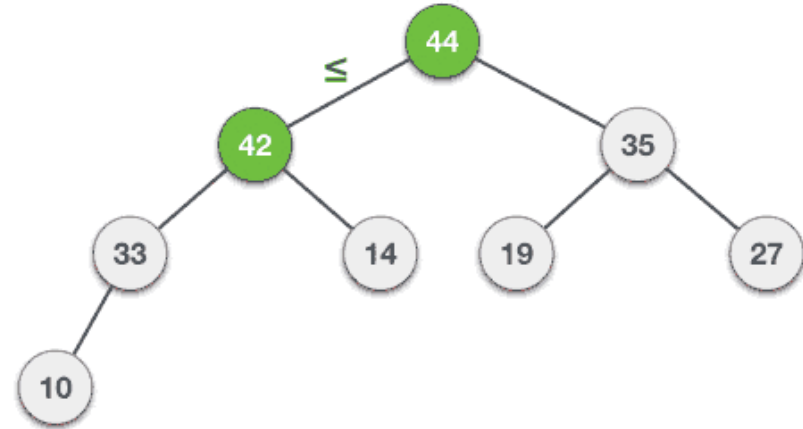
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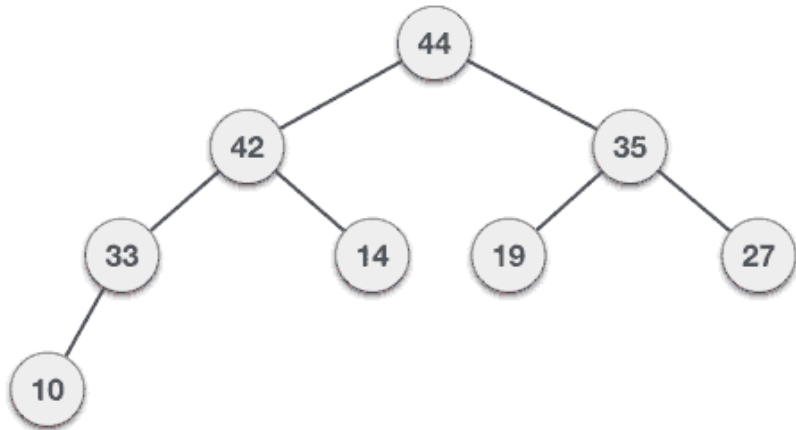
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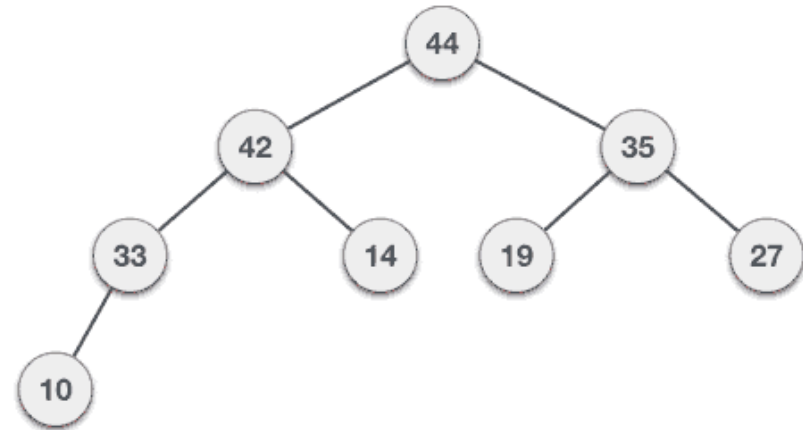
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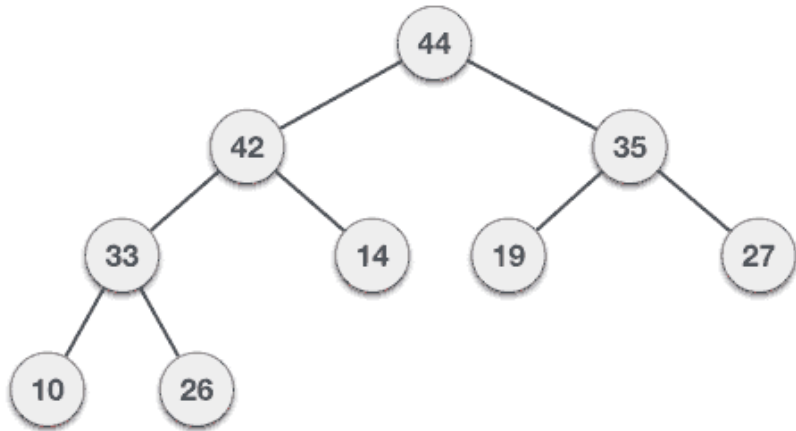
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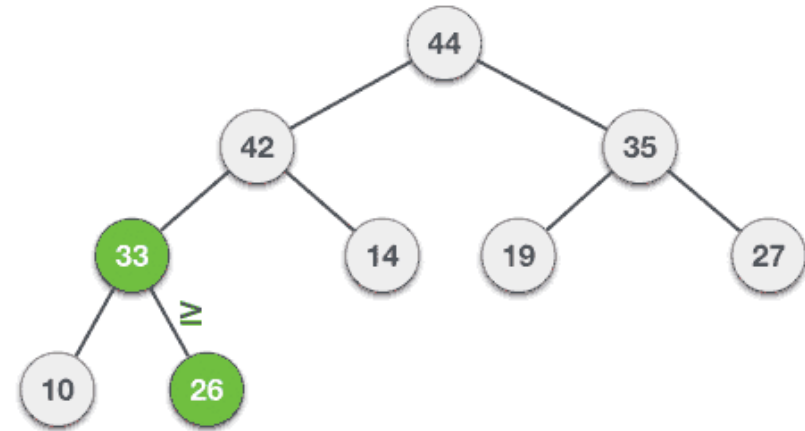
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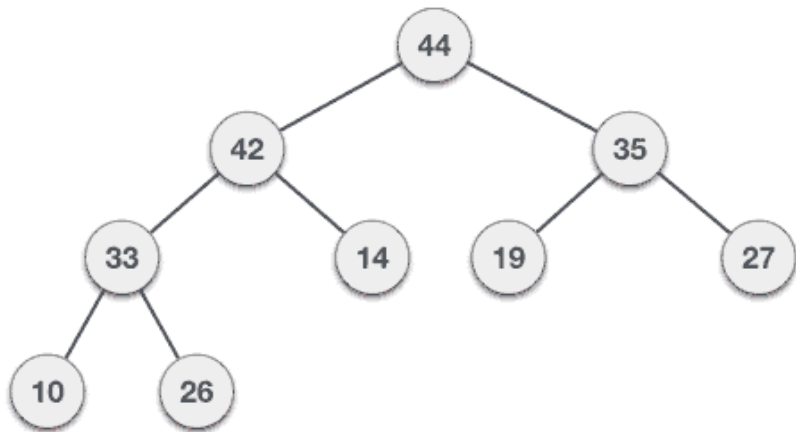
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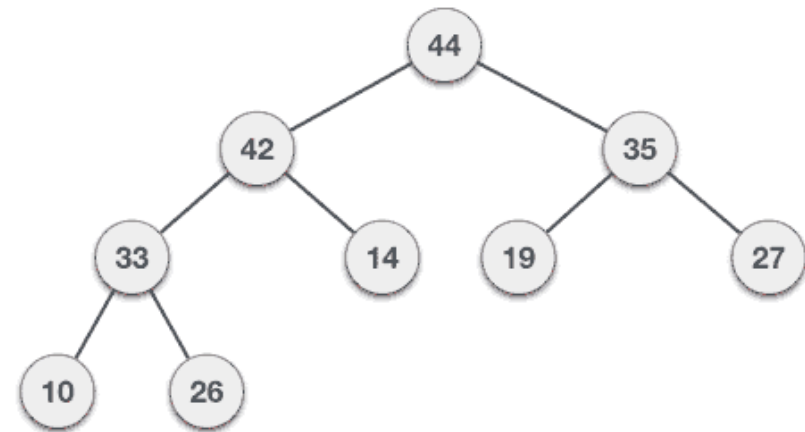
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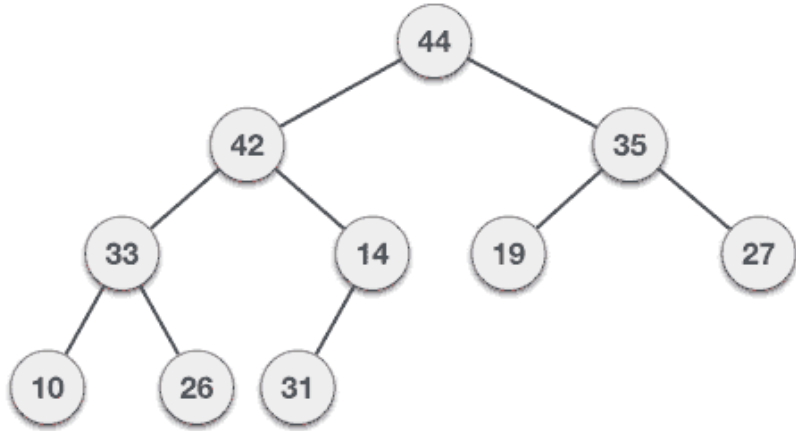
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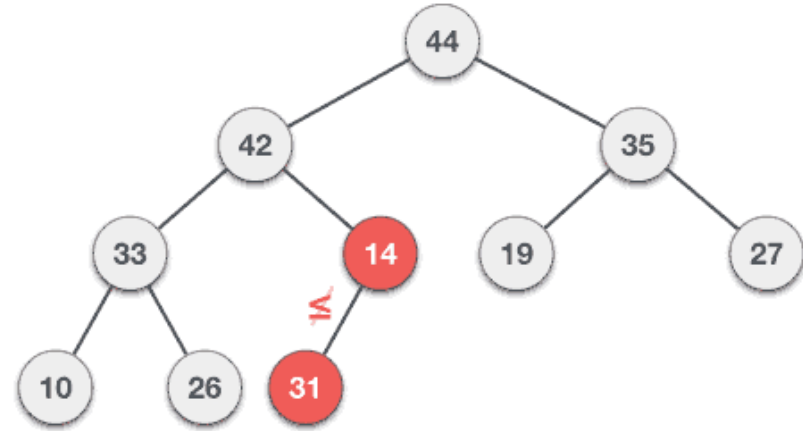
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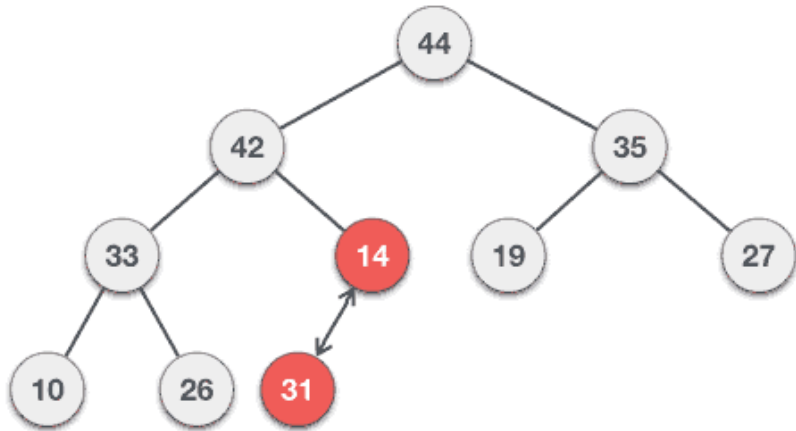
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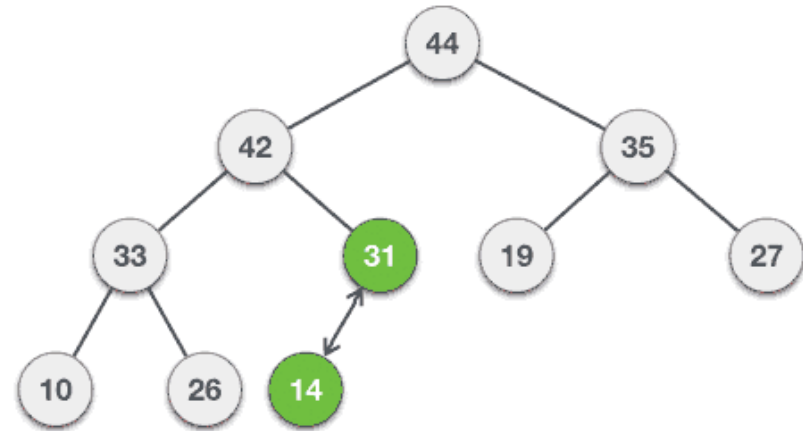
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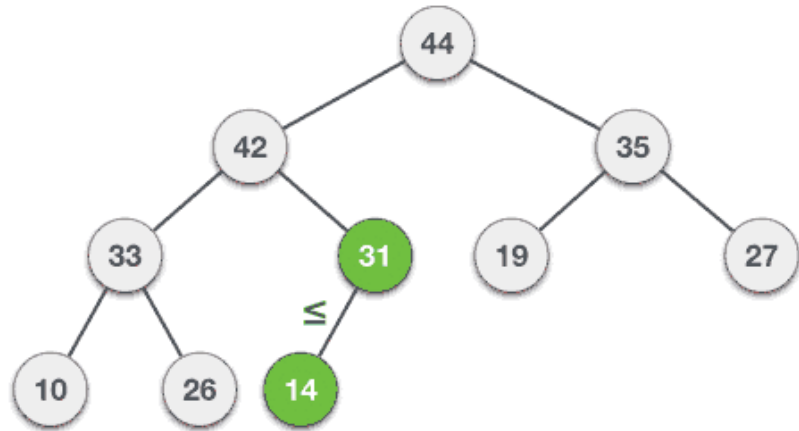
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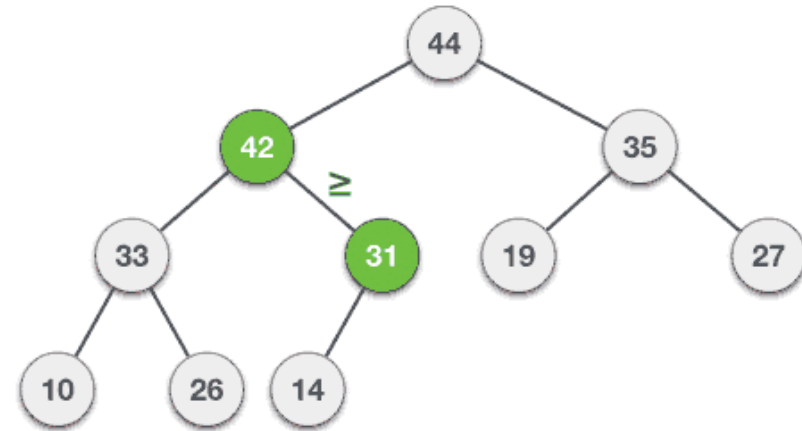
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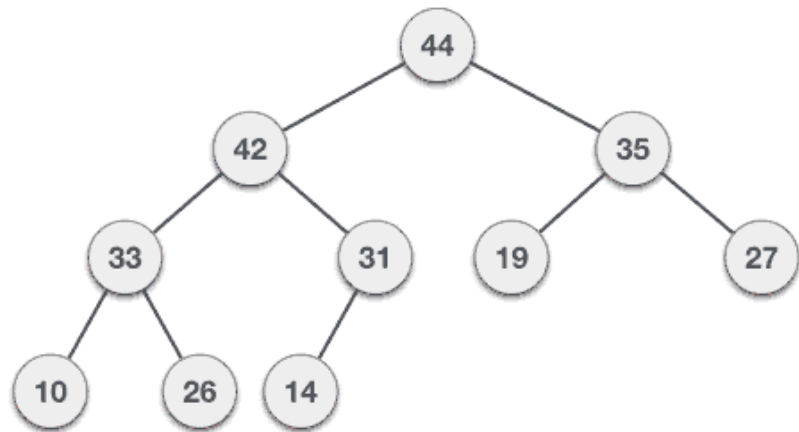
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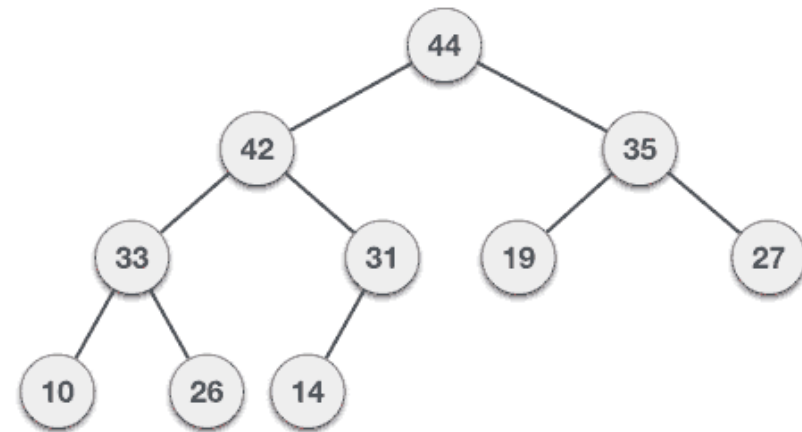
Max Heap – Add Example

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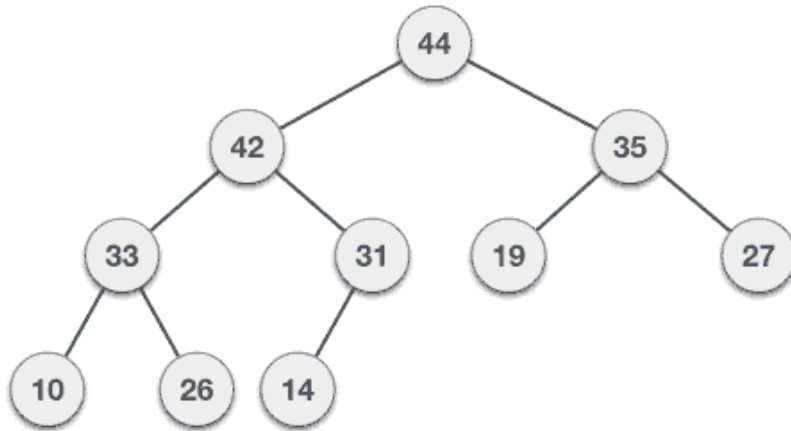
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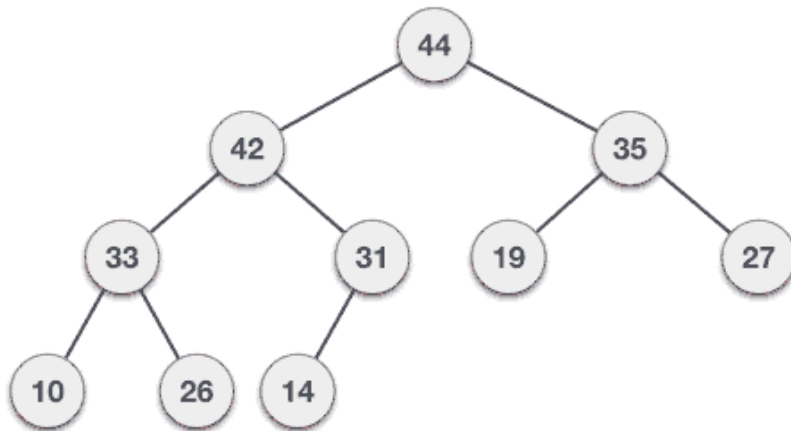


Max Heap – Deletion

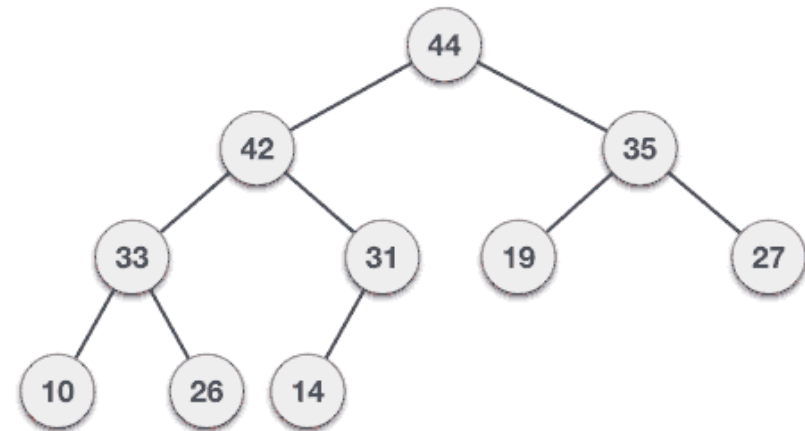
1. Remove root node.
2. Move the last element of last level to root.
3. Compare the value of this child node with its parent.
4. If value of parent is less than child, then swap them.
5. Repeat step 3 & 4 until Heap property holds.



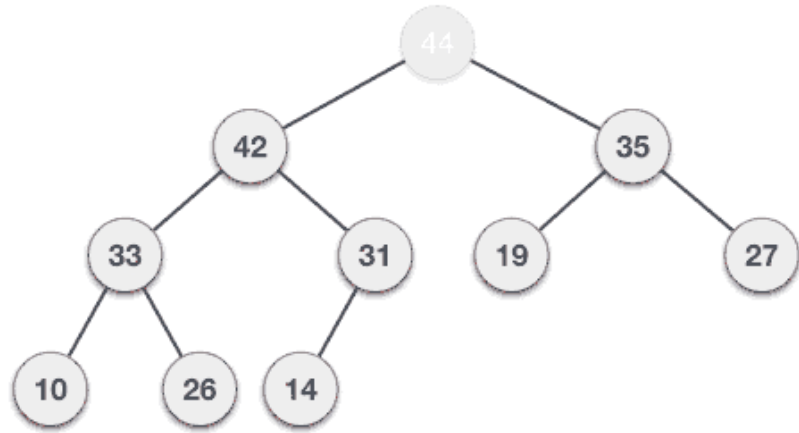
Max Heap – Deletion Example



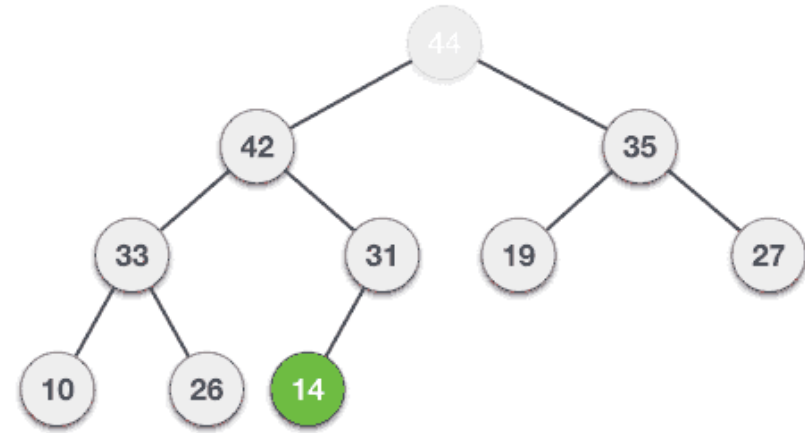
Max Heap – Deletion Example



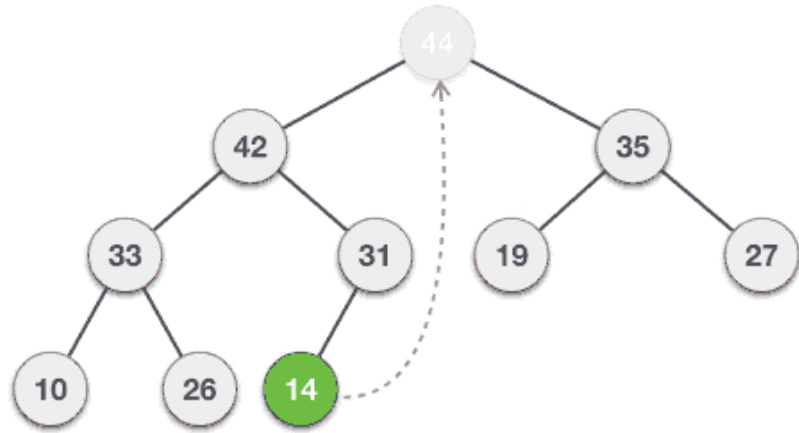
Max Heap – Deletion Example



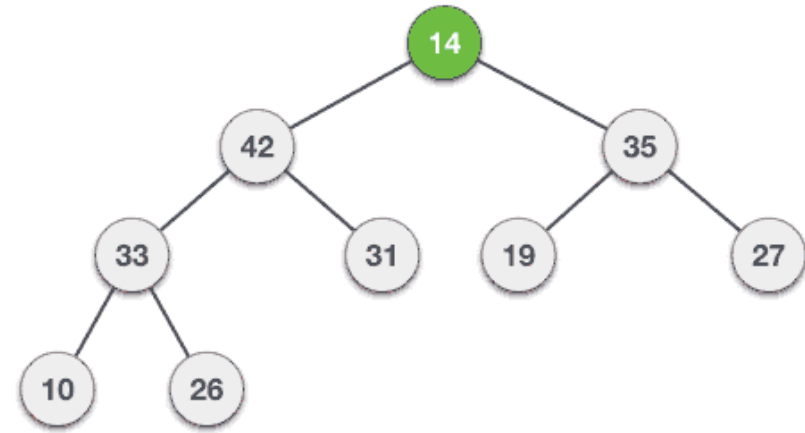
Max Heap – Deletion Example



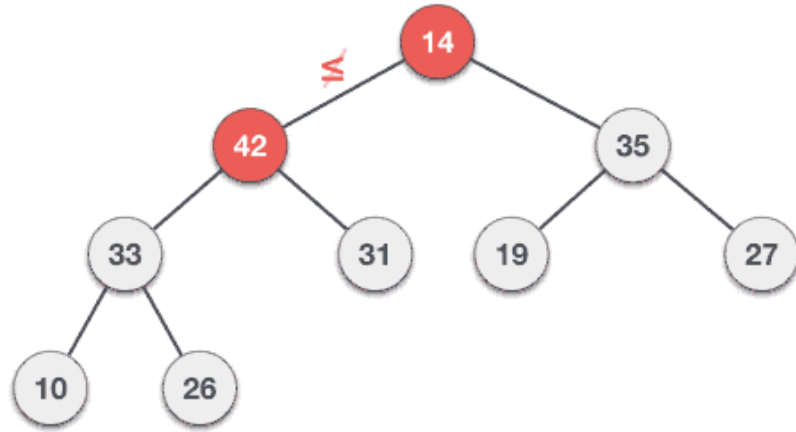
Max Heap – Deletion Example



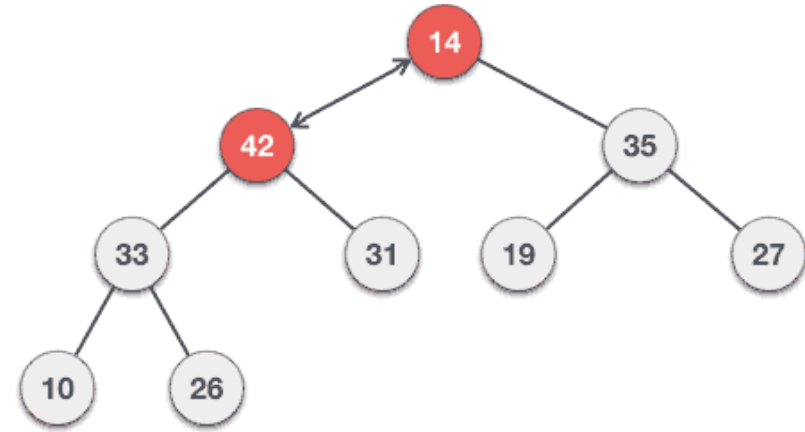
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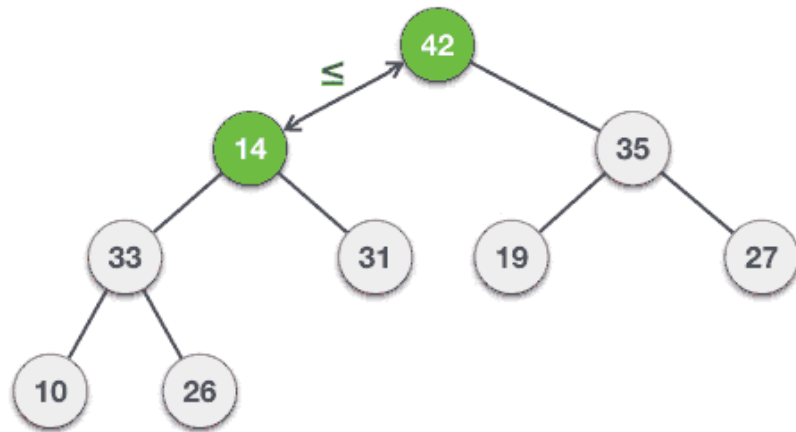
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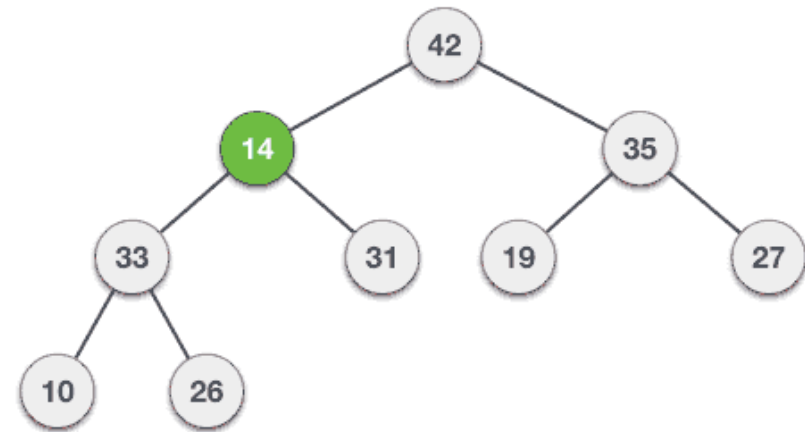
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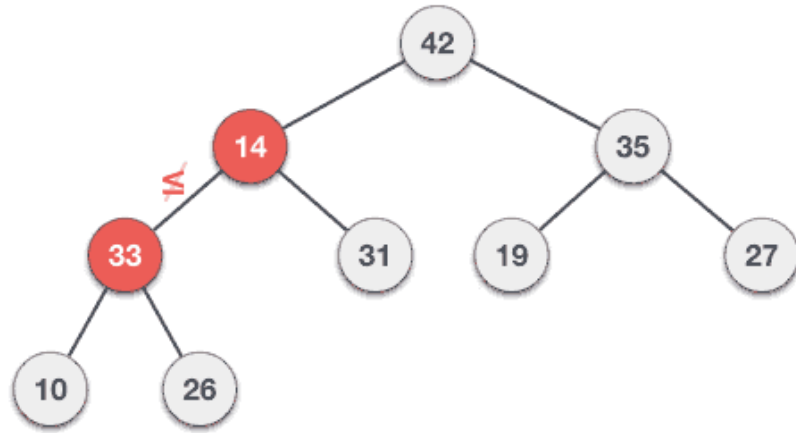
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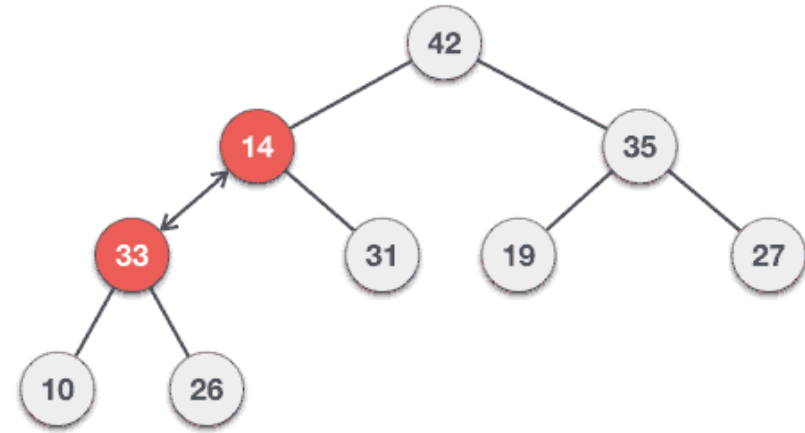
Max Heap – Deletion Example



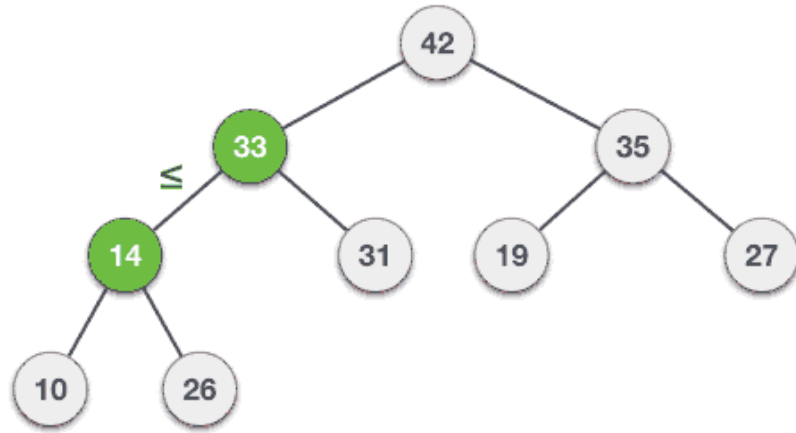
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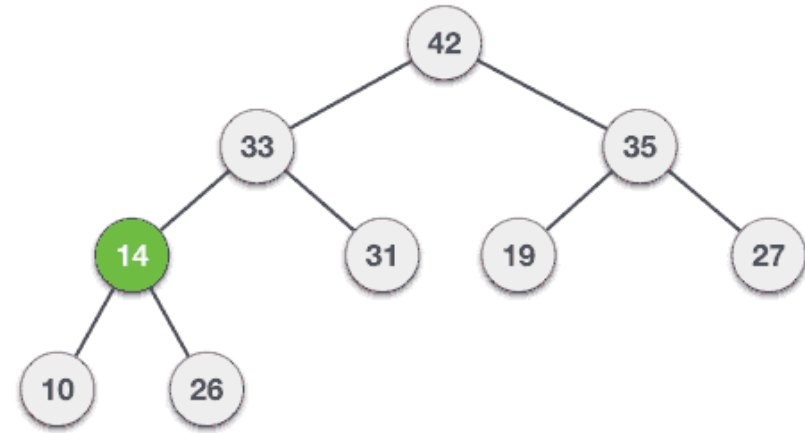
Max Heap – Deletion Example



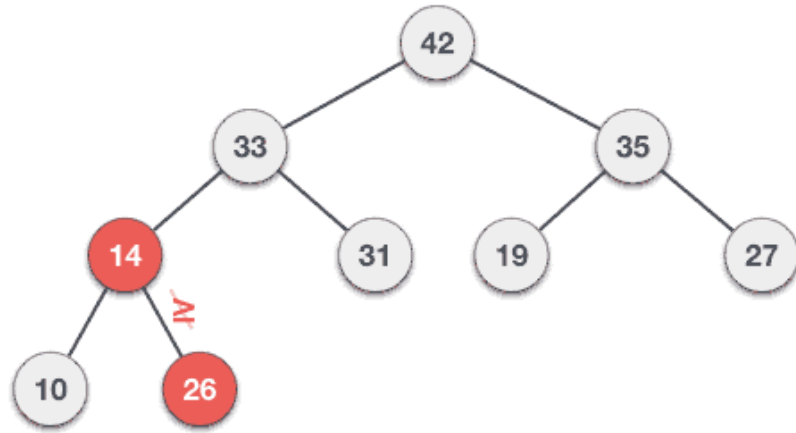
Max Heap – Deletion Example



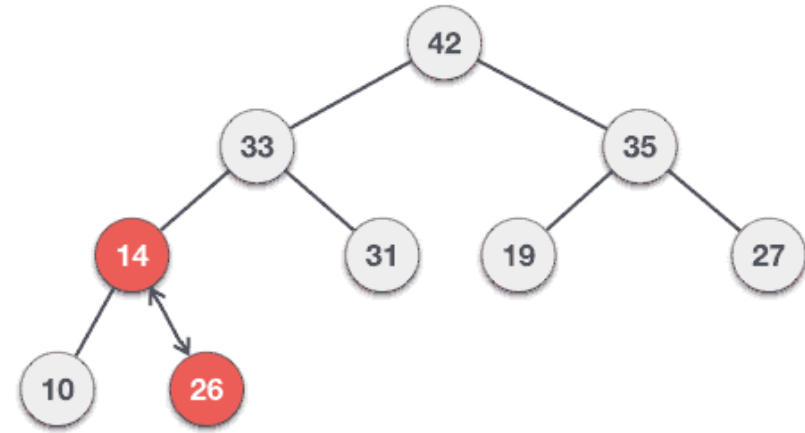
Max Heap – Deletion Example



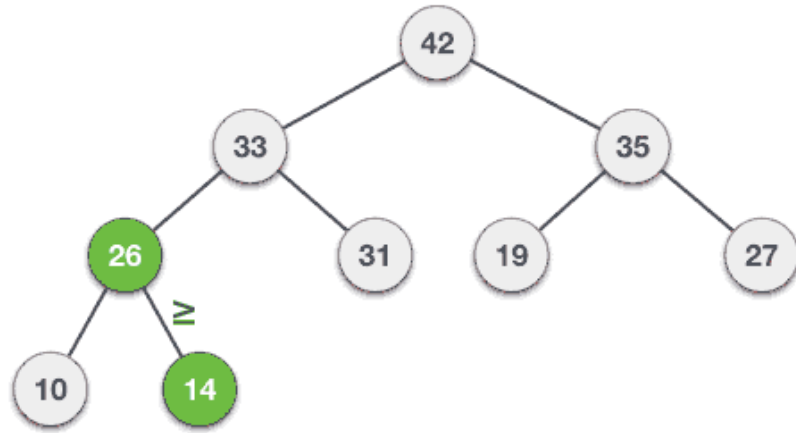
Max Heap – Deletion Example



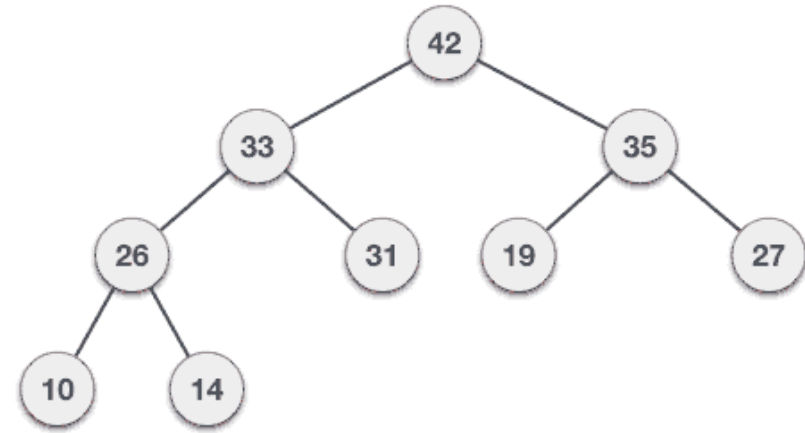
Max Heap – Deletion Example



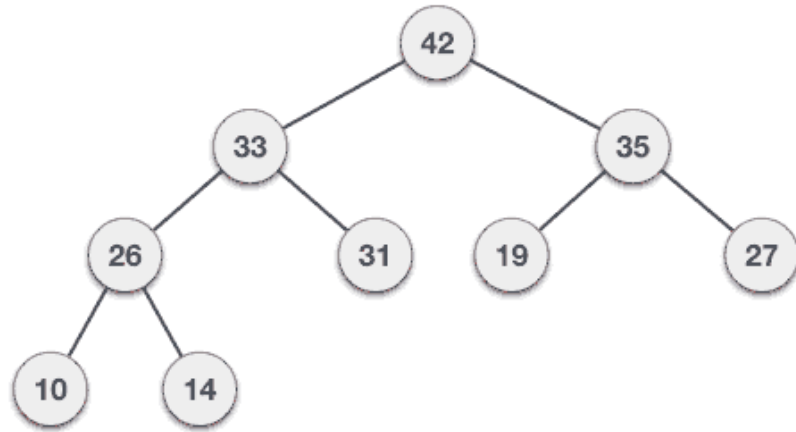
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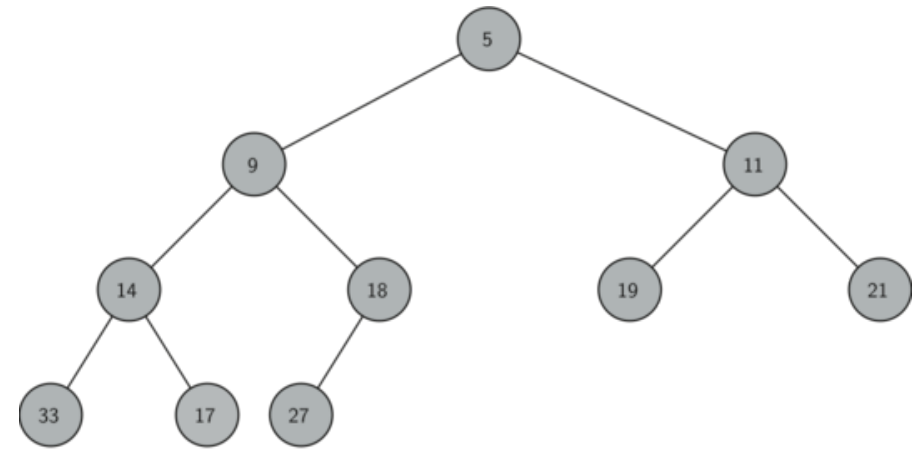
Max Heap – Deletion Example



Max Heap – Deletion Example



Heap Implementation using Arrays



0	5	9	11	14	18	19	21	33	17	27	
0	1	2	3	4	5	6	7	8	9	10	11



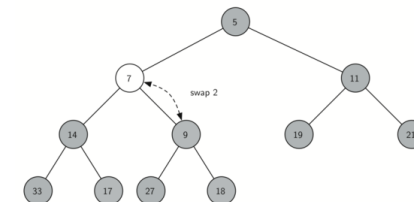
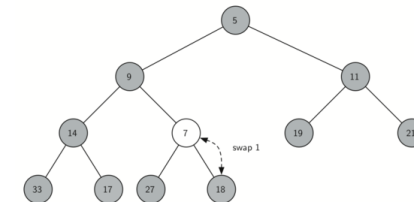
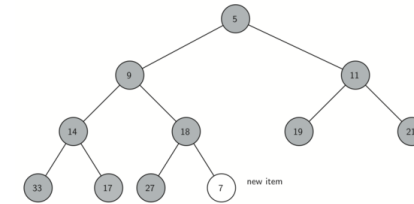
Binary Heap

```

1 class BinHeap:
2     def __init__(self):
3         self.heapList = [0]
4         self.currentSize = 0
    
```



Binary Heap – Insertion

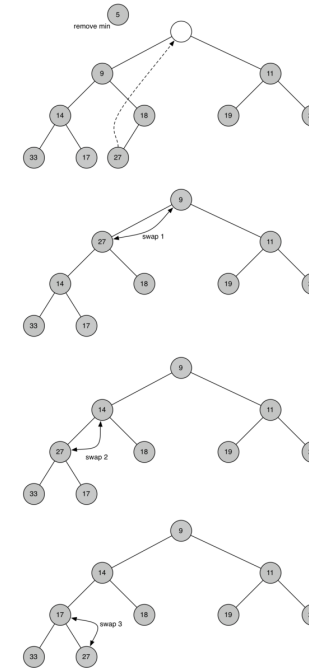


Binary Heap – Insertion

```
1 def percUp(self, i):
2     while i // 2 > 0:
3         if self.heapList[i] < self.heapList[i // 2]:
4             tmp = self.heapList[i // 2]
5             self.heapList[i // 2] = self.heapList[i]
6             self.heapList[i] = tmp
7         i = i // 2
8
9 def insert(self, k):
10    self.heapList.append(k)
11    self.currentSize = self.currentSize + 1
12    self.percUp(self.currentSize)
```



Binary Heap – Remove



Binary Heap – Remove

```
1 def percDown(self, i):
2     while (i * 2) <= self.currentSize:
3         mc = self.minChild(i)
4         if self.heapList[i] > self.heapList[mc]:
5             tmp = self.heapList[i]
6             self.heapList[i] = self.heapList[mc]
7             self.heapList[mc] = tmp
8         i = mc
9
10 def minChild(self, i):
11     if i * 2 + 1 > self.currentSize:
12         return i * 2
13     else:
14         if self.heapList[i*2] < self.heapList[i*2+1]:
15             return i * 2
16         else:
17             return i * 2 + 1
```

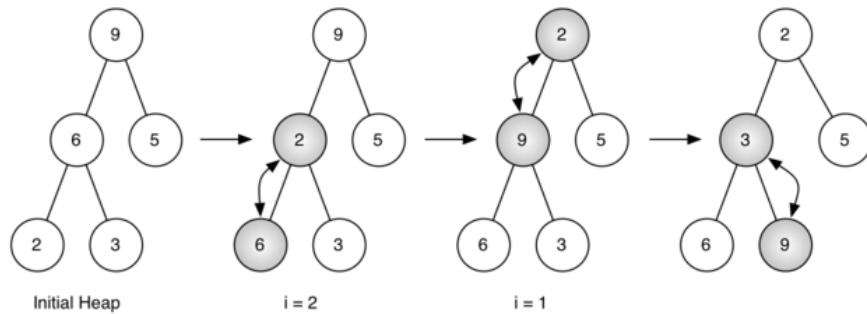


Binary Heap – Remove

```
1 def delMin(self):
2     retval = self.heapList[1]
3     self.heapList[1] = self.heapList[self.currentSize]
4     self.currentSize = self.currentSize - 1
5     self.heapList.pop()
6     self.percDown(1)
7     return retval
```



Binary Heap – Build from List



Binary Heap – Build from List

```
1 def buildHeap(self, alist):
2     i = len(alist) // 2
3     self.currentSize = len(alist)
4     self.heapList = [0] + alist[:]
5     while (i > 0):
6         self.percDown(i)
7         i = i - 1
```

3rd Assignment

- ▶ Generate a random list of integers (where $n = \{10, 100, 1000, 10000, \dots\}$) and do a benchmark analysis for each one:
- ▶ Implement a new class that keeps track of the maximum and minimum number in a list of numbers.
 - ▶ Use a python list based on Quicksort, BubbleSort
 - ▶ Use a Binary Tree
 - ▶ Use a python Heap
- ▶ Use `timeit` to measure execution time
- ▶ Produce plots for each of the above results visualizing the performance of the different algorithms
- ▶ Email ichatz@diag.uniroma1.it
Subject: [PCS2] Homework 3
A link to a github repository with your python code.
- ▶ **Deadline: 21/December/2018, 23:59**